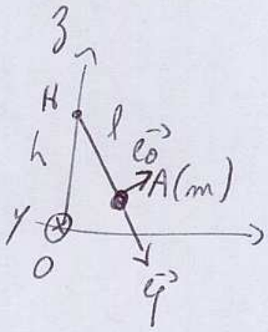

A. Pendule simple

PARTIE I :

1. $\vec{P} = m\vec{g} = -mg\vec{e_z} = mg\cos\theta\,\vec{e_\rho} - mg\sin\theta\,\vec{e_\theta}$

$\vec{T} = -T\vec{e_\rho}$

2. $m\vec{a}_{A/R} = \vec{P} + \vec{T} \Rightarrow \begin{cases} -ml\dot\theta^2 = mg\cos\theta - T \\ ml\ddot\theta = -mg\sin\theta \end{cases}$

$\hookrightarrow \boxed{\ddot\theta + \frac{g}{l}\sin\theta = 0}$

3. $T_0 = \frac{2\pi}{\omega_0}$ avec $\omega_0^2 = \frac{g}{l}$

$\Rightarrow T_0 = 2\pi\sqrt{\frac{l}{g}}$. A.N. : $T_0 = 0{,}78\,s$

PARTIE II :

4. a) $\vec{F_{ie}} = -m\vec{a_e} = -m\vec{a}_{O_1/R} = -ma_0\vec{e_x} = -ma_0(\sin\theta\,\vec{e_\rho} + \cos\theta\,\vec{e_\theta}$

$\vec{F_{ic}} = -m\vec{a_c} = \vec{0}$

b) $m\vec{a}_{A/R} = \sum\vec{F} + \vec{F_{ie}}$

$\Leftrightarrow \begin{cases} -ml\dot\theta^2 = mg\cos\theta - T - ma_0\sin\theta \\ ml\ddot\theta = -mg\sin\theta - ma_0\cos\theta \end{cases}$

$\Rightarrow \boxed{\ddot\theta + \frac{g}{l}\sin\theta + \frac{a_0}{l}\cos\theta = 0}$

θ petit $\Rightarrow \sin\theta \simeq \theta$, $\cos\theta \simeq 1$ $\Rightarrow \boxed{\ddot\theta + \frac{g}{l}\theta + \frac{a_0}{l} = 0}$

c) $\vec{\mathcal{M}}_H(\vec{P}+\vec{T}+\vec{F}_{ie}) = \vec{\mathcal{M}}_H(\vec{P}) + \vec{\mathcal{M}}_H(\vec{F}_{ie}) = \vec{HA}\wedge\vec{P} + \vec{HA}\wedge\vec{F}_{ie}$

$= l\vec{e_\rho}\wedge(-mg\sin\theta\,\vec{e_\theta} - ma_0\cos\theta\,\vec{e_\theta})$

$= (mgl\sin\theta + ma_0 l\cos\theta)\,\vec{e_y}$.

d) $\vec{L_H} = \vec{HA}\wedge m\vec{v}_{A/R} = l\vec{e_\rho}\wedge ml\dot{\theta}\vec{e_\theta} = -ml^2\dot{\theta}\vec{e_y}$

e) $\left.\dfrac{d\vec{L_H}}{dt}\right|_R = \sum\vec{\mathcal{M}}_H(\vec{F}) \Rightarrow -ml^2\ddot{\theta} = mgl\sin\theta + ma_0 l\cos\theta$

$$\boxed{\begin{array}{l}\Rightarrow \ddot{\theta} + \frac{g}{l}\sin\theta + \frac{a_0}{l}\cos\theta = 0 \\ \Rightarrow \ddot{\theta} + \frac{g}{l}\theta + \frac{a_0}{l} = 0 \quad \text{si } \theta \ll 1\end{array}}$$

f) $\ddot{\theta} + \frac{g}{l}\cancel{\theta}\,\theta + \frac{a_0}{l} = 0 \Rightarrow T_1 = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{l}{g}} = T_0$.

$\Rightarrow T_0 = T_1$ car force constante ne fait que déplacer la position d'équilibre.

B) Energie d'une particule sur une ellipse

1a) $$\vec{r} = a\cos(\omega t)\,\vec{i} + b\sin(\omega t)\,\vec{j}$$

$$\frac{d\vec{r}}{dt} = -a\omega\sin(\omega t)\,\vec{i} + b\omega\cos(\omega t)\,\vec{j}$$

$$\frac{d^2\vec{r}}{dt^2} = -a\omega^2\cos(\omega t)\,\vec{i} - b\omega^2\sin(\omega t)\,\vec{j} = -\omega^2\vec{r}$$

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} = -m\omega^2 r\,\frac{\vec{r}}{r} = -m\omega^2 r\,\vec{e_r}$$

force centrale → cs → dérive d'1 $\mathcal{E}_{pot}$

$$\vec{F} = -m\omega^2 r\,\vec{e_r} = -\vec{grad}\,\mathcal{E}_p = -\frac{\partial \mathcal{E}_p}{\partial r}\vec{e_r}$$

soit $\dfrac{\partial \mathcal{E}_p}{\partial r} = m\omega^2 r \Rightarrow \mathcal{E}_p(r) = \dfrac{m\omega^2 r^2}{2}$

1b) $$W_{\vec{F}\,M_1\to M_2} = -\frac{m\omega^2}{2}(r_2^2 - r_1^2)$$

2) Cs de l'$\mathcal{E}_m$

$$\mathcal{E}_m = \mathcal{E}_k + \mathcal{E}_p = \frac{1}{2}m\left(\frac{d\vec{r}}{dt}\right)^2 + \frac{m\omega^2 r^2}{2}$$

$$= \frac{1}{2}m\left(a^2\omega^2\sin^2(\omega t) + b^2\omega^2\cos^2(\omega t)\right)$$

$$+ \frac{m\omega^2}{2}\left(a^2\cos^2(\omega t) + b^2\sin^2(\omega t)\right)$$

$$= \frac{m\omega^2}{2}(a^2+b^2) = \text{cste}$$ et indépendant du temps.

3) cdt $\sin^2(\omega t) = \cos^2(\omega t) \Rightarrow \omega t = \pi/4 + k\,\pi/2$ k entier

$$= (1+2k)\pi/4$$

$$t = (1+2k)\frac{\pi}{4\omega}\ ;$$ k entier